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Handout - Calculating Pitch (Teacher's Guide)

An instrument can produce different sound wave frequencies based on variables related to its physical construction. In music, we refer to sound wave frequencies as pitch. Pitch tells us how "high" or "low" a note sounds.

The frequency of the soundwave, or pitch of a note, that a string instrument will play can be calculated by using the following equation:

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

where

f_1 = fundamental frequency in Hertz (Hz)

T = tension of the string in Newtons (N)

μ = mass per unit length of the string in kilograms per meter (kg/m)

L = length of the string in meters (m)

Guided Practice

Below, calculate the pitch of a string that is 2 meters long, a mass of 0.120 kg/m, and is under 800 N of tension.

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

$$f_1 = \frac{\sqrt{\frac{800\ N}{0.120\ kg/m}}}{2(2m)}$$

$$f_1 = \frac{\sqrt{\frac{800}{0.120}}}{4}$$

$$f_1 = \frac{\sqrt{6666.6667}}{4}$$

$$f_1 = \frac{81.64965809}{4}$$

$$f_1 = 20.4124\ Hz$$

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Independent Work

Background Information: The beam is made up of 13 bass piano strings. A typical bass string on a grand piano is 5' 2'' (1.5748 meters) long with a mass of 0.120 kilograms per meter. Mickey Hart tunes the beam differently depending on where he is performing. For "D" tuning, the beam's fundamental low string has a frequency of 36.7081 hertz; for "A" tuning, the beam's fundamental low string has a frequency of 55.0000 hertz; and for "E" tuning, the beam's fundamental low string has a frequency of 41.2034 hertz.

Problem 1

You are Mickey Hart's technical manager for his upcoming tour, and part of your job is to tune the beam for his upcoming shows. Use the above information to determine the amount of tension required to tune the beam to different pitches.

A. Calculate the amount of tension that needs to be applied to the string to tune the string on the beam to a low "D":

$$36.7081\ Hz = \frac{\sqrt{\frac{T}{.120\ kg/m}}}{2(1.5748\ m)}$$

$$36.7081 = \frac{\sqrt{\frac{T}{.120}}}{3.1496\ m}$$

$$115.6158^2 = \sqrt{\frac{T}{0.120}}^2$$

$$13367.0206 = \frac{T}{0.120}$$

$$T = 1604.0425\ N$$

Note on optional differentiation: *The formula for fundamental frequency can be rearranged to highlight the quantity of interest, tension (T). Ask students to show how they would use their knowledge of inverse operations to make T the subject of the equation. You may scaffold this for students that need more support with step-by-step guidance or pose this as a challenge to students that need an extension.*

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L}$$

1. Multiply both sides by 2L; 2. Square both sides; 3. Multiply both sides by μ

$$T = (\mu)(2L)^2(F)^2$$

Differentiation (T as the subject of the equation):

$T = (0.120\ kg/m)(2*1.5748\ m)^2(36.7081\ Hz)^2$

$T = (0.120)(3.1496)^2(36.7081)^2$

$T = (0.120)(9.91998016)(1347.48460561)$

$T = 1604.0425\ N$

B. Does the tension on the string need to increase or decrease when tuning from "D" to "A"? Prove your answer mathematically and explain in words.

$$55.0000\ Hz = \frac{\sqrt{\frac{T}{.120\ kg/m}}}{2(1.5748\ m)}$$

$$55.0000 = \frac{\sqrt{\frac{T}{.120}}}{3.1496}$$

$$173.228^2 = \sqrt{\frac{T}{0.120}}^2$$

$$30007.939984 = \frac{T}{0.120}$$

$$T = 3600.9528\ N$$

Differentiation (T as the subject of the equation):

$T = (0.120\ kg/m)(2*1.5748\ m)^2(55.0000\ Hz)^2$

$T = (0.120)(9.91998016)(3025)$

$T = 3600.9528\ N$

If the length and mass per unit length of the string remained the same, the amount of tension on the string would increase to create a higher pitch.

C. Does the tension on the string need to increase or decrease when tuning from "A" to "E"? Prove your answer mathematically and explain in words.

$$41.2034 = \frac{\sqrt{\frac{T}{.120\ kg/m}}}{2(1.5748\ m)}$$

$$41.2034 = \frac{\sqrt{\frac{T}{.120}}}{3.1496}$$

$$129.7742^2 = \sqrt{\frac{T}{0.120}}^2$$

$$16841.3430 = \frac{T}{0.120}$$

$$T = 2020.9612\ N$$

Differentiation (T as the subject of the equation):

$T = (0.120\ kg/m)(2*1.5748\ m)^2(41.2034\ Hz)^2$

$T = (0.120)(9.91998016)(1697.72017156)$

$T = 2020.9621\ N$

If the length and mass per unit length of the string remained the same, the amount of tension on the string would decrease to create a lower pitch.

Problem 2

Mickey Hart is curious if he could build a smaller version of the beam that is half the length but still capable of producing the same low pitch.

A. If the strings on the beam are half their current length and the tension on the string and the mass per unit length of the string remain the same, how would the pitch of the sound produced compare to a low "D" (36.7081 HZ)? Prove your answer mathematically and explain in words.

$$F_1 = \frac{\sqrt{\frac{1604.0425}{.120\ kg/m}}}{2(0.7874\ m)}$$

$$F_1 = \frac{\sqrt{\frac{1604.0425}{.120}}}{1.5748}$$

$$F_1 = \frac{\sqrt{13367.02083}}{1.5748}$$

$$F_1 = \frac{115.615832956}{1.5748}$$

$$F_1 = 73.4162\ Hz$$

Cutting the strings in half will produce a sound with a higher pitch.

B. If you want to get back to the same low "D" tuning at 36.7081 HZ, would the mass of the string need to increase or decrease? Prove your answer mathematically and explain in words.

$$36.7081\ Hz = \frac{\sqrt{\frac{1604.0425\ N}{\mu}}}{2(0.7874\ m)}$$

$$36.7081 = \frac{\sqrt{\frac{1604.0425}{\mu}}}{1.5748}$$

$$57.80791588^2 = \sqrt{\frac{1604.0425}{\mu}}^2$$

$$3341.75513839 = \frac{1604.0425}{\mu}$$

$$\mu = 0.480\ kg/m$$

Differentiation (T as the subject of the equation):

$1604.0425\ N = (\mu)(2*0.7874\ m)^2(36.7081\ Hz)^2$

$1604.0425 = (\mu)(1.5748)^2(36.7081)^2$

$1604.0425 = (\mu)(2.480)(1347.4846)$

$1604.0425 = (\mu)(3341.761808)$

$\mu = 0.480 kg/m$

If the tension and desired frequency remained the same, the mass per unit length of the string would need to increase from 0.120 kg/m to 0.480 kg/m

C. The beam uses low "D" piano strings, which have a mass of 120 kg/m. These strings are some of the lowest strings manufactured. Based on your calculations on the preceding question, do you think it is practical to build an instrument half the size as the beam that produces the same pitches? Why or why not?

In order to construct an instrument that is half the size of the beam and produces the same pitches, the mass of the string would need to increase from 0.120 kg/m to 0.480 kg/m. It is unlikely that we would be able to use a string that is four times the mass of the lowest strings currently manufactured.

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Problem 3

When Mickey Hart plays the beam, it often goes through a custom-built "Reality Amplifier" which digitally lowers the sound of the instrument by an octave. Originally, there was a discussion of creating a version of the beam without the "Reality Amplifier" capable of producing a pitch that low, but it was deemed impractical.

A. If a string is lowered an octave, it's fundamental frequency is halved. If the beam is tuned to "D" (36.7081 HZ), what frequency would be produced through the Reality Amplifier?

The note an octave below "D" produced by the Reality Amplifier would have a fundamental frequency of 18.351 HZ.

B. Assuming the mass and tension of the "D" string on the beam is the same, what would the length of the beam have to be to produce a note an octave below "D" (36.7081 HZ)? Prove mathematically and explain why constructing such an instrument would be impractical:

$$18.3541\ Hz = \frac{\sqrt{\frac{1604.0425\ N}{.120\ kg/m}}}{2(L)}$$

$$18.3541\ Hz = \frac{\sqrt{13367.0201}}{2(L)}$$

$$18.3541\ Hz = \frac{115.6158}{2(L)}$$

$$36.7082(L) = 115.6158$$

$$L = 3.1500\ m$$

In order to produce this note, the strings on the beam would need to double in length from 1.5748 m (5' 2") to 3.1500 (10" 4').

Group Work

Discuss with your group what you have learned about how tension, length, and mass per unit length affect pitch. Summarize your learning below:

As the tension on the string increases, the pitch increases
As the tension on the string decreases, the pitch increases

If the length of the string increases, the pitch decreases
If the length of the string decreases, the pitch increases

If the mass per unit length of the string increases, the pitch decreases
If the mass per unit length of the string decrease, the pitch increases